B PHYSICS AND CP VIOLATION

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These lectures present the phenomenology of B meson decays and their impact on our understanding of CP violation in the quark sector, with an emphasis on measurements made at the e^+e^- B factories. Some of the relevant theoretical ideas such as the Operator Product Expansion and Heavy Quark Symmetry are introduced, and applications to the determination of CKM matrix elements given. The phenomenon of B flavor oscillations is reviewed, and the mechanisms for and current status of CP violation in the B system is given. The status of rare B decays is also discussed.

1. Flavor Physics with B Mesons

B decays provide a sensitive probe of the physics of quark mixing, described by the unitary Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix in the Standard Model (SM). The mixing of the weak and mass eigenstates of the quarks provides a rich phenomenology and gives a viable mechanism for the non-conservation of CP symmetry in the decays of certain hadrons. CP asymmetries in B decays can be large and allow a determination of the magnitude of the irreducible phase in the CKM matrix. The pattern of CP asymmetries observed in B decays can be compared with the detailed prediction of these asymmetries in the SM in an effort to tease out evidence of new physics (NP). The study of B decays allows direct measurement of the magnitudes of the elements $|V_{ub}|$ and $|V_{cb}|$ of the CKM matrix. The large mass of the top quark allows information on $|V_{td}|$ and $|V_{ts}|$ to be extracted via higher order processes like $B^0\overline{B}^0$ oscillations and decays involving loop diagrams. These loop decays have accessible branching fractions (as large

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as 10^{-4}) and allow sensitive searches for physics beyond the SM, as the effect of new particles or couplings in the internal loops can manifest itself in modifications to the rates for these decays.

In addition to probing the flavor sector of the SM, B decays provide a laboratory for testing our understanding of QCD. The scale of the short-distance physics (e.g. weak b quark decay) is in the perturbative regime of QCD while the formation of final state hadrons and the binding of the b quark to the valence anti-quark is clearly non-perturbative. Powerful theoretical tools have been developed to systematically address the disparate scales. The Operator Product Expansion (OPE) together with Heavy Quark Symmetry allow perturbative calculations to be combined with non-perturbative matrix elements, and provide relationships amongst the non-perturbative matrix elements contributing to different processes. This allows some non-perturbative quantities to be determined experimentally, and leads to the vibrant interplay between experiment and theory that has characterized this area of research in recent years.

2. Quark mixing

The mixing between the quark mass eigenstates and their weak interaction eigenstates, parameterized in the unitary 3×3 CKM matrix, is responsible for flavor oscillations in the neutral B and K mesons and leads to an irreducible source of CP violation in the SM through the non-trivial phase of the CKM matrix. The CKM matrix can be parameterized by 3 real angles and one imaginary phase. The presence of this irreducible phase is an unavoidable consequence of 3 generation mixing.[a] Many parameterizations of the CKM matrix have been suggested. One choice in widespread use is in terms of angles θ_{12}, θ_{13}, θ_{23} and phase δ (see [1]). The magnitudes of the elements in the CKM matrix decrease sharply as one moves away from the diagonal, suggesting a parameterization[2] in terms of powers of λ, the sine of the the Cabibbo angle. An improved version[3] of this "Wolfenstein parameterization" will be used here. The starting point is to let the parameters λ, A, ρ and η satisfy the relations $\lambda = |V_{us}|$, $A\lambda^2 = |V_{cb}|$ and $A\lambda^3(\rho - i\eta) = |V_{ub}|$ and to write the remaining elements in terms of these four parameters, expanded in powers of λ. Given that $\lambda = 0.2196 \pm 0.0026$[1], highly accurate approximate parameterizations can be obtained keeping only the first one or two terms in the expansion. The matrix to order λ^5 is given here.

[a] The number of irreducible phases for N-generation mixing is $(N-2)(N-1)/2$.

$$\begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda+\frac{1}{2}A^2\lambda^5\left[1-2(\rho+i\eta)\right] & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2 \\ A\lambda^3\left[1-(\rho+i\eta)(1-\frac{1}{2}\lambda^2)\right] & -A\lambda^2+A(\frac{1}{2}-\rho-i\eta)\lambda^4 & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix} \quad (1)$$

This parameterization has several nice features. The smallness of the off-diagonal elements is taken up by powers of λ, leaving the parameters A, ρ and η of order unity. It also makes clear the near equality of the elements $|V_{cb}|$ and $|V_{ts}|$, namely $|V_{ts}/V_{cb}| = 1 + \mathcal{O}(\lambda^2)$.

The relations dictated by unitarity allow a convenient geometrical representation of the CKM parameters. The product of any row (column) of the matrix times the complex conjugate of any other row (column) results in three complex numbers that sum to zero, and can be drawn as a triangle in the complex plane. There are three such independent triangles. Two of the three have one side much shorter than the others (i.e. have one side that is proportional to a higher power of λ than are the others), but the remaining triangle, formed by multiplying the first column by the complex conjugate of the third column, has all sides of order λ^3. It is this triangle that is usually discussed when considering the impact of experimental measurements on the parameters of the CKM matrix. These unitarity relations need to be verified experimentally; a violation of unitarity would point to new physics (e.g. a fourth generation, in which case the 3×3 submatrix need not be unitary).

The unitarity triangle of interest, namely $V_{ud}V^*_{ub}+V_{cd}V^*_{cb}+V_{td}V^*_{tb} = 0$, is usually rescaled by dividing through by $V_{cd}V^*_{cb}$, giving a triangle whose base has unit length and lies along the x axis and whose apex is at the point $\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}} = 1 + \frac{V_{td}V^*_{tb}}{V_{cd}V^*_{cb}}$. Casting these relations in terms of the parameters λ, A, ρ and η and defining $\overline{\rho} = (1-\lambda^2/2)\rho$ and $\overline{\eta} = (1-\lambda^2/2)\eta$, the apex of the triangle is at $(\overline{\rho},\overline{\eta})$ up to corrections of order λ^2. By dividing out V_{cb} we largely eliminate dependence on the parameter A, which is in any case relatively well known[1] ($A = 0.85 \pm 0.04$). The sides and angles of the

unitarity triangle can be expressed as

$$R_u = \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \simeq -\sqrt{\overline{\rho}^2 + \overline{\eta}^2}e^{i\gamma} \quad (2)$$

$$R_t = \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} \simeq -\sqrt{(1-\overline{\rho})^2 + \overline{\eta}^2}e^{i\beta} \quad (3)$$

$$\gamma = \arg V_{ub}^* \quad (4)$$

$$\beta = \arg V_{td} \quad (5)$$

$$\alpha = \pi - \beta - \gamma \quad (6)$$

The angles α, β and γ are also known as ϕ_2, ϕ_1 and ϕ_3, respectively. Figure 1 shows the constraints on the unitarity triangle given in Ref. [1].

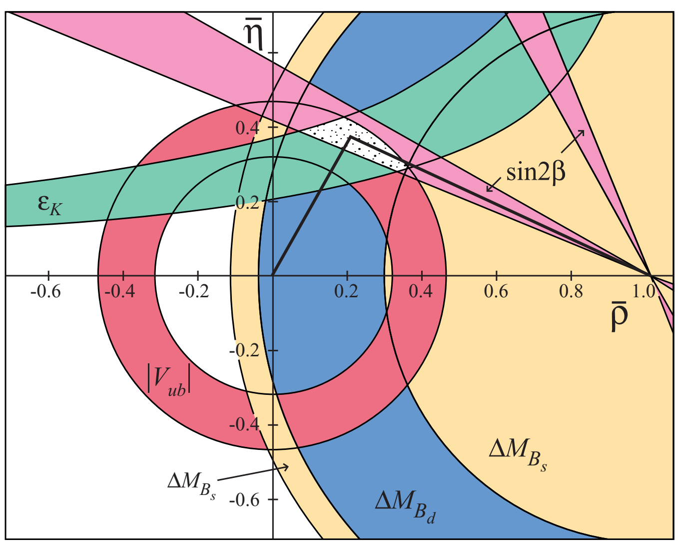

Figure 1. Constraints in the $\overline{\rho}-\overline{\eta}$ plane from measurements in the B meson and neutral K systems.

Measurements of $|V_{ub}|/|V_{cb}|$ constrain the length of R_u and constrain $(\overline{\rho}, \overline{\eta})$ to lie in an annulus centered on the origin. These measurements are improving as more data and additional theoretical insights are brought to bear. The bands emanating from the point (1,0) correspond to the measurement of $\sin 2\beta$, the result of the impressive initial success of the B-factory program. These measurements already give some of the most precise information about the unitarity triangle and are perhaps an order of magnitude

away from being limited by systematics. The B system also allows access to the length of R_t through measurements of $B^0\overline{B}^0$ oscillations, which give indirect sensitivity to $|V_{td}|$ and $|V_{ts}|$. The determination of R_t is dominated by theoretical uncertainties. Measurements of B_s^0 oscillations and of the ratio of $b \to d\gamma$ to $b \to s\gamma$ decays should allow improved determinations of R_t. While B_s^0 oscillations are not accessible at e^+e^- B factories, there are good prospects for this measurement at high energy hadron collider experiments. Measurement of the CKM-suppressed $b \to d\gamma$ decays will require significantly larger data sets than are currently available.

3. B factory basics

The e^+e^- B factories operate at the $\Upsilon(4S)$ resonance, a quasi-bound $b\overline{b}$ state just above the threshold for production of B^+B^- or $B^0\overline{B}^0$ pairs. The width of the $\Upsilon(4S)$ is comparable to the beam energy spread of e^+e^- colliders, making the peak cross-section a weak function of the machine parameters. The cross-section at the peak is about 1.1 nb, to be compared with the underlying $q\overline{q}(q = u, d, s, c)$ cross-section of ~ 3.5 nb. The $\Upsilon(4S)$ is expected to decay with a branching fraction of 0.5 to each of B^+B^- and $B^0\overline{B}^0$; measurements are consistent with this expectation, but have a large uncertainty[1] ($\pm 14\%$ on the ratio of the two branching fractions). The mass difference between the $\Upsilon(4S)$ and a pair of B mesons is about 20 MeV, so no additional particles (apart from low-energy radiated photons) are produced, and the final state B mesons have one unit of orbital angular momentum and small velocity ($\beta \simeq 0.06$) in the $\Upsilon(4S)$ rest frame.

The cross-section for b hadron production at hadron colliders is much higher—typically by a factor of 100 or more, depending on the collider energy and the acceptance in pseudo-rapidity. In addition, B_s^0 and b baryons are also produced. However, the fraction of events containing b hadrons is much lower (a few per mil) and only those events leaving a clean trigger signature are accessible. In the short term the most important B physics measurements that will be made at the hadron colliders are the B_s^0 oscillation frequency and CP asymmetries in B_s^0 decay.

3.1. *Asymmetric e^+e^- B factories*

The asymmetric B factories designed to study CP asymmetries in B decays collide low energy positiron beams (3.1-3.5 GeV) with higher energy electron beams (9.0-8.0 GeV) and give the center-of-mass of the collision a boost of $\beta\gamma \simeq 0.5$ along the beam axis. As a result the distance between the

decay points of the two B mesons, whose lifetimes are ~ 1.6 ps, is typically 250 μm. This is critical in measuring CP asymmetries that arise in the interference between amplitudes for $B^0\overline{B}^0$ mixing and B decay, since the time integral of these CP asymmetries vanishes. The required luminosity is set by the scale of the branching fractions for B^0 decay to experimentally accessible CP eigenstates and the need to determine the flavor of the other B in the event and the time difference between the two B decays. This leads to a requirement of 30 fb^{-1}/year, or a luminosity of at least $\mathcal{L} = 3 \times 10^{33}$ cm^{-2} s^{-1}. The B factory detectors must provide good particle identification over the full momentum range, excellent vertex resolution, and good efficiency and resolution for charged tracks and photons.

Two B factories exceeding these minimum requirements have been operating since 1999: the Belle detector at the KEK-B accelerator at KEK and the BaBar detector at the PEP-II accelerator at SLAC. Both facilities have been operating well, providing unprecedented luminosities (as of February, 2003):

	KEK-B / Belle	PEP-II / BaBar
$\mathcal{L}_{\rm max}$ (10^{33}cm^{-2}s^{-1})	8.3	4.8
best day (pb^{-1})	434	303
total (fb^{-1})	106	96

The BaBar and Belle detectors are broadly similar: each has a silicon vertex detector surrounded by a wire tracking chamber with a Helium-based gas, particle identification, a CsI(Tl) calorimeter, a superconducting coil and detectors interspersed with the iron flux return to measure K_L^0 and identify muons. They differ in their silicon detectors, where Belle uses 3 layers at small radii (3.0-5.8 cm) while BaBar adds two layers at large radii (up to 12.7 and 14.6 cm); in each case the wire tracking device starts a few centimeters beyond the last silicon layer. The biggest difference is in the technology used for particle identification. While both detectors exploit dE/dx in the tracking chambers to identify low momentum hadrons, Belle uses a combination of time-of-flight and aerogel threshold Cherenkov counters. BaBar has a different strategy, using long quartz bars to generate Cherenkov light from passing particles and to transport it via total internal reflection to a water-filled torus where the Cherenkov angle is measured. The performance of each detector is similar in most respects.

4. B hadron decay

The weak decay of a free b quark is completely analogous to muon decay. While there are no free quarks, this is a useful starting point for understanding B hadron decay. The B meson lifetime is relatively large (~ 1.6 ps), much longer than the lighter D mesons or tau lepton, due to the smallness of $|V_{cb}|$. Semileptonic decays are prominent, with branching fractions of about 10.5% each for semi-electronic and semi-muonic decays and about 2.5% for semi-tauonic decays. These decays contain a single hadronic current and are more tractable theoretically than fully hadronic final states, which make up the bulk of the B decay rate. Purely leptonic decays are helicity suppressed (less so for tau modes) and require either CKM-suppressed $b \to u$ transitions or Flavor-Changing Neutral-Currents (FCNC), forbidden at tree level in the SM, leading to branching fractions[4] $\mathcal{B}(B^+ \to \tau^+\nu_\tau) \sim 6 \times 10^{-5}$, $\mathcal{B}(B^0 \to \tau^+\tau^-) \sim 3 \times 10^{-8}$.

The long B lifetime, along with the large top quark mass which breaks the GIM mechanism, give radiative "penguin" FCNC decays like $b \to s\gamma$ and $b \to s\ell\overline{\ell}$ branching fractions in the $10^{-4} - 10^{-6}$ range, making them accessible. These same factors result in large $B^0\overline{B}^0$ mixing through second order weak processes involving box diagrams with virtual W and top particles, and make rare B decays a fruitful ground for searching for physics beyond the SM.

4.1. *Theoretical picture of B decay*

The material in this subsection is covered in more detail in several excellent review articles.[5] The free quark picture is a useful starting point, but the impact of the strong interactions binding the b quark in the B hadron must be addressed to achieve a quantitative understanding of B decay. Early attempts to do this involved models that gave the b quark an r.m.s. "Fermi momentum" to account for bound state effects. While these models improved the understanding of some observables, like the lepton energy spectrum in semileptonic decay, they did not provide a quantitative assessment of the theoretical uncertainties. Since the early 1990s new theoretical methods have been used based on a systematic separation of short-distance and long-distance scales through the Operator Product Expansion (OPE). The OPE formalism is used to develop effective field theories where the short distance behaviour is integrated out of the theory. A scale μ is defined to separate the long- and short-distance regimes. The choice of μ is in principle arbitrary, since no observables can depend on it *if the calculation is carried*

out to all orders. In practice, μ is chosen to satisfy $\Lambda_{\rm QCD} \ll \mu \ll M_W$. The result of integrating out the short-distance heavy particle fields is a non-local action that is then expanded in a series of local operators of increasing dimension whose (Wilson) coefficients contain the short-distance physics. Perturbative corrections (e.g. for hard gluon emission) to the short-distance physics are incorporated using renormalization-group methods.[6] B decay amplitudes are expressed as

$$A(B \to F) = \langle F|H_{\rm eff}|B\rangle = \sum_i C_i(\mu)\langle F|Q_i(\mu)|B\rangle \tag{7}$$

The Wilson coefficients $C_i(\mu)$ typically include leading-log or next-to-leading-log corrections. The sum involves increasing powers of the heavy quark mass, required to offset the increasing dimensions of the non-perturbative operators. This suppression of higher-dimension operators is central to our ability to make quantitative predictions for B decays. At present only terms of order $1/m_b^2$ or $1/m_b^3$ nnare considered, leaving only a modest number of non-perturbative matrix elements to determine (either experimentally or through non-perturbative calculational techniques).

A major step forward in the understanding of B decays was the recognition of heavy quark symmetry. The scale $\Lambda_{\rm QCD} \sim 0.2$ GeV at which QCD becomes non-perturbative is small compared to the heavy quark mass m_Q. As a result the gluons binding the heavy quark and light spectator are too soft to probe the quantum numbers—mass, spin, flavor—of the heavy quark. In the limit $m_Q \to \infty$ the heavy quark degrees of freedom decouple completely from the light degrees of freedom, resulting in a spin-flavor SU($2N_h$) symmetry, where N_h is the number of heavy quark flavors[b] This symmetry has an analogue in atomic physics, where the nucleus acts as a static source of electric charge and where nuclear properties decouple from the degrees of freedom associated with the electrons; to first approximation different isotopes or nuclear spin states of an element have the same chemistry. In heavy-light systems, the heavy quark acts as a static source of color charge.

Heavy quark symmetry forms the basis of an effective field theory of QCD, namely Heavy Quark Effective Theory (HQET). The key observation is that in the heavy quark limit the velocities of the heavy quark and the hadron containing it are equal: $p_Q = m_Q v + k$, where p_Q is the 4-vector of the heavy quark, v is the 4-velocity of the hadron and k is the residual

[b] Effectively two, since the top quark decays before hadronizing.

momentum, whose components are small compared to m_Q. The degrees of freedom associated with energetic ($\mathcal{O}(2m_Q)$) fluctuations of the heavy quark field are integrated out, resulting in an effective Lagrangian

$$L_{\rm eff} = \overline{h}_v iv \cdot D h_v + \frac{1}{2m_Q}\overline{h}_v (i\not{D}_\perp)^2 h_v + \frac{g_s}{4m_Q}\overline{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v + \mathcal{O}(m_Q^{-2}) \quad (8)$$

where $h_v(x) = e^{im_Q v\cdot x}\frac{1+\not{v}}{2}Q(x)$ is the upper 2 components of the heavy quark Dirac spinor, the lower components having been integrated out of the theory. The first term is all that remains in the limit $m_Q \to \infty$, and is manifestly invariant under SU($2N_h$). The second term is the kinetic energy operator O_K for the residual motion of the heavy quark, and the third term gives the operator O_G for the interaction of the heavy quark spin with the color-magnetic field. The matrix elements associated with these operators are non-perturbative, but can be related to measurable quantities.

The non-perturbative parameters at lowest order are $\lambda_1 = \langle Q|O_K|Q\rangle/2m_Q$ and $\lambda_2 = -\langle Q|O_G|Q\rangle/6m_Q$. The mass of a heavy meson can be written

$$m_{H_Q} = m_Q + \overline{\Lambda} + \frac{-\lambda_1 + 2\left[J(J+1) - \frac{3}{2}\right]\lambda_2}{2m_Q} + \mathcal{O}\left(m_Q^{-2}\right) \quad (9)$$

The parameter $\overline{\Lambda}$ arises from the light quark degrees of freedom, and is defined by $\overline{\Lambda} = \lim_{m_Q\to\infty}(m_{H_Q} - m_Q)$. In the heavy quark limit all systems with the same light quark degrees of freedom should have the same $\overline{\Lambda}$. It can be verified that this gives a good description of SU(3)$_{\rm flavor}$ breaking ($m(B_s) - m(B_d) \simeq m(D_s) - m(D_d)$). The mass splitting between the vector and pseudo-scalar mesons determines λ_2 to be approximately 0.12 GeV:

$$m^2(B^*) - m^2(B) = 0.49 \text{ GeV}^2 = 4\lambda_2 + \mathcal{O}(m_b^{-1}) \quad (10)$$

$$m^2(D^*) - m^2(D) = 0.55 \text{ GeV}^2 = 4\lambda_2 + \mathcal{O}(m_c^{-1}) \quad (11)$$

4.2. *Exclusive semileptonic decays and* $|V_{cb}|$

Heavy quark effective theory is clearly a powerful tool in understanding transitions between two heavy-light systems, since the light degrees of freedom don't see the change in heavy quark flavor or spin in the heavy quark limit. An area of great practical importance is the determination of $|V_{cb}|$ from exclusive semileptonic decays like $B \to \overline{D}^* \ell^+ \nu$. In the heavy quark limit the form factor for the decay can depend only on the product of 4-velocities of the initial and final mesons, $w = v_B \cdot v_{D^*}$. This universal form factor is known as the Isgur-Wise function[7] $\xi(w)$. The form of the function

is not specified in HQET, but its normalization is unity at the "zero-recoil" point, $\xi(1) = 1$, where the D^* meson is stationary in the B meson rest frame and the light degrees of freedom are blind to the change in heavy quark properties.

One of the striking predictions of HQET is that the four independent form factors in a general $P \to V\ell\overline{\nu}$ transition are all related to the Isgur-Wise function:

$$h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w)$$
$$h_{A_2}(w) = \left[\frac{2m_B m_{D^*}(1+w)}{(m_B + m_{D^*})^2}\right]\xi(w) \to 0 \text{ as } m_Q \to \infty$$

where h_V is the form factor for the vector current and h_{A_1}, h_{A_2} and h_{A_3} are the form factors associated with the axial-vector current. These relations can be tested experimentally.

The normalization of the physical form factor at zero recoil differs from unity due to QCD radiative corrections and heavy quark symmetry-breaking corrections. This leads to

$$\mathcal{F}(w) = \xi(w) + \mathcal{O}(\alpha_s(m_Q)) + \mathcal{O}\left((\Lambda_{\mathrm{QCD}}/m_Q)^2\right) + ... \tag{12}$$
$$\mathcal{F}(1) = \eta_A\left(1 + C\frac{\Lambda^2_{\mathrm{QCD}}}{m_Q^2}\right) \tag{13}$$

where η_A encorporates the QCD correction[8] and the absence of a correction at order $\Lambda_{\mathrm{QCD}}/m_Q$ is known as Luke's theorem.[9] The value $\mathcal{F}(1)$ must be calculated using non-perturbative techniques such as Lattice QCD or QCD sum rules. These lead to the currently accepted value[10] of 0.91 ± 0.04. This rather precise prediction can be combined with an experimental measurement of the decay rate for $B \to \overline{D}^* \ell^+\nu$ to determine $|V_{cb}|^2$. The measurement is complicated by the need to extrapolate the differential decay rate $d\Gamma/dw$ to the point $w = 1$, since $d\Gamma/dw$ vanishes there. A further complication comes from the fact that the transition pion from the $D^* \to D$ decay has very low momentum in the B frame for $w \simeq 1$. Nevertheless, the measured experimental rates give $\mathcal{F}(1)|V_{cb}| = (38.3 \pm 1.0) \times 10^{-3}$ from which a precise value of $|V_{cb}|$ is obtained:[1]

$$|V_{cb}| = (42.1 \pm 1.1 \pm 1.9) \times 10^{-3} \tag{14}$$

Similar measurements can be made for $B \to \overline{D}\ell^+\nu$. The theoretical situation here is less favorable, since Luke's theorem does not prevent $\Lambda_{\mathrm{QCD}}/m_Q$ corrections in this case, and the experimental situation is complicated by feed-down from $B \to \overline{D}^*\ell^+\nu$ decays. The ratio of form factors

at zero recoil can be measured and compared with predictions:

$$\frac{\mathcal{G}(1)}{\mathcal{F}(1)} = 1.08 \pm 0.06 \text{ (theory)}$$
$$= 1.08 \pm 0.09 \text{ (experiment)}$$

where $\mathcal{G}(w)$ is the form factor in $B \to \overline{D}\ell^+\nu$ decay. The tests of the HQET predictions for form factors in semileptonic B decays to charm are nearly at the point of testing the symmetry breaking terms, and are improving.

4.3. *Inclusive semileptonic decays*

The OPE and HQET formalism can be used to study inclusive semileptonic decays. Bound state effects can be accounted for in a systematic expansion in terms of α_S and $1/m_b$. To do this, however, one must introduce a new assumption, namely that the (long distance) process of forming color singlet final state hadrons does not change the rates calculated at the quark level. This assumption goes under the name of quark-hadron duality. While it has been demonstrated to hold under certain conditions — e.g. in the cross-section for $e^+e^- \to$hadrons and in tau decays to hadrons — it also clearly breaks down in regions where the density of color-singlet final states is small or zero, e.g. for $m_{\text{had}} < 2m_\pi$. While duality violations in inclusive rates are expected to be small, their level is hard to quantify, and they can be important when a severely restricted phase space is examined (e.g. when selecting $b \to u\ell\overline{\nu}$ decays by requiring that the charged lepton momentum exceeds the kinematic endpoint for leptons from $b \to c\ell\overline{\nu}$ decays).

The semileptonic decay rate in the Heavy Quark Expansion (HQE) is

$$\Gamma(B \to X\ell^+\nu) = \frac{G_F^2 m_b^5}{192\pi^3}\left[1 + C_1\frac{\alpha_s(m_b)}{\pi} + ... + \frac{-\lambda_1 - 9\lambda_2}{2m_b^2} + ...\right] \quad (15)$$

where the non-perturbative matrix elements λ_1 and λ_2 are familiar from HQET. Note the absence of a $1/m_b$ correction term; this allows the term in brackets to be computed to a precision of about 5%. The dependence on m_b^5 can result in large uncertainties in the theoretical prediction. These have been brought under control by using the upsilon expansion,[11] in which (ignoring the subtleties) one-half the mass of the $\Upsilon(1S)$ is substituted for m_b. The resulting theoretical expression for extracting $|V_{ub}|$ from the corresponding inclusive semileptonic decay width is[12]

$$|V_{ub}| = (3.87 \pm 0.10 \pm 0.10) \times 10^{-3} \times \sqrt{\frac{\Gamma(\overline{B} \to X_u\ell\overline{\nu})}{1.0\text{ns}^{-1}}} \quad (16)$$

A similar expression relates $|V_{cb}|$ to $\Gamma(\overline{B} \to X_c \ell \overline{\nu})$:

$$|V_{cb}| = (44.5 \pm 0.8 \pm 0.7 \pm 0.5) \times 10^{-3} \times \sqrt{\frac{\Gamma(\overline{B} \to X_c \ell \overline{\nu})}{65.6\text{ns}^{-1}}} \tag{17}$$

Measuring the semileptonic width for $b \to c\ell\nu$ transitions is straightforward; up to small corrections one just measures the semileptonic branching fractions and lifetimes of B mesons and sets $\Gamma_{\rm sl} = \mathcal{B}_{\rm sl}/\tau_B$. The momentum spectrum of leptons from B decay is fairly stiff, while leptons from other processes (most notably $b \to c \to \ell$ decays) are softer. The formula given above can then be used to extract $|V_{cb}|$ with small theoretical uncertainty. Proceeding in this manner with the present experimental information[1] on $\tau(B^0) = 1542 \pm 16$ fs, $\tau(B^+) = 1674 \pm 18$ fs and $\mathcal{B}(\overline{B} \to X_c \ell \overline{\nu}) = (10.38 \pm 0.32)\%$ gives[1]

$$|V_{cb}| = (40.4 \pm 0.5_{\rm exp} \pm 0.5_{\rm non-pert} \pm 0.8_{\rm pert}) \times 10^{-3} \tag{18}$$

This determination is fully consistent with the value derived from exclusive semileptonic decays.

The determination of $|V_{ub}|$ from inclusive semileptonic decays is more challenging due to the large background from the CKM-favored $b \to c$ transitions. The first measurements of $|V_{ub}|$ came from the endpoint of the lepton momentum spectrum, where a small fraction ($\sim 10\%$) of the leptons from $b \to u$ transitions lie above the endpoint for leptons from $b \to c$ transitions. The experimental signal in this region is very robust, but the limited acceptance results in large theoretical uncertainties in extracting $|V_{ub}|$. These uncertainties arise because of limited knowledge of the so-called shape function, i.e. the distribution of the (virtual) b quark mass in the B meson, which affects the kinematic distributions of the final state particles and therefore changes the fraction f_u of $b \to u$ decays above the minimum accepted lepton momentum.

One means of reducing the theoretical uncertainty is to obtain information on the shape function from other B decays. The easiest method conceptually is to examine the photon energy spectrum in the B rest frame for $b \to s\gamma$ decays.[13] Since the photon does not undergo strong interactactions it probes the b quark properties, with $m_b = 2E_\gamma$ at lowest order. The first moment of this spectrum is essentially the mean b quark mass (or, equivalently, $\overline{\Lambda}$) while the second moment is essentially $-\lambda_1$. Similar use can be made[14] of the recoiling W in semileptonic $b \to u$ decays, where $m_b = E_W + |\vec{p}_W|$, but requires the reconstruction of the neutrino momentum. Measurements of other moments in semileptonic decays of

both $b \to u$ and $b \to c$ (e.g. of E_ℓ, $m^2_{\rm had}$...) also give information on the non-perturbative parameters $\overline{\Lambda}$ and λ_1 and can be used to constrain the range of variation that must be considered in assessing the theoretical error due to the acceptance cuts.

Another approach to reducing the theoretical uncertainty in $|V_{ub}|$ from inclusive semileptonic decays is to measure more than just the charged lepton. Setting aside the experimental difficulties, a measurement of the mass of the recoiling hadron allows a much greater fraction of the $b \to u$ final states to be accepted, namely those with invariant mass $m_{\rm had} < m_D$, thereby reducing theoretical uncertainties.[15] Measuring the invariant mass of the lepton and neutrino (q^2) and requiring $q^2 > (m_B - m_D)^2$, while having a lower acceptance than a cut on $m_{\rm had}$, results in a similar theoretical uncertainty.[14] Both of these approaches are in progress at the B factories. These approaches should yield $|V_{ub}|$ with uncertainties of 10% or less in the near future.

Exclusive charmless semileptonic decays ($B \to \pi\ell^+\nu$, etc.) provide an independent means of determining $|V_{ub}|$. The experimental measurements of these decays are improving. At present the leading uncertainties come from theoretical calculations and models of form factors. There is an expectation that lattice QCD calculations of these form factors will eventually allow $|V_{ub}|$ to be extracted with uncertainties of less than 10%, providing an independent test of the $|V_{ub}|$ determined from inclusive semileptonic decays.

5. $B^0\overline{B}^0$ oscillations

The material developed in the next two sections is covered in greater depth in several excellent reviews.[19,20,16] Quark flavor is not conserved in weak interactions. As a result, transitions are possible between neutral mesons and their antiparticles. These transitions result in the decay eigenstates of the particle–anti-particle system being distinct from their mass eigenstates. In systems where the weak decay of the mesons is suppressed (e.g. by small CKM elements) and the Δ(flavor)$= 2$ transitions between particle and anti-particle are enhanced (due to the large top mass and favorable CKM elements) the decay eigenstates can be dramatically different from the mass eigenstates, resulting in the spectacular phenomenon of flavor oscillations. The flavor oscillations first observed in the neutral K system result in the striking lifetime difference between the two decay eigenstates and the phenomenon of regeneration. In neutral B mesons, the lifetime differences are

small as is the branching fraction to eigenstates of CP. As a result, oscillations are observed by studying the time evolution of the flavor composition (b or $\overline{b}$) of weak decays to flavor eigenstates.

The neutral B mesons form a 2-state system, with the flavor eigenstates denoted by

$$|B^0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |\overline{B}^0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{19}$$

The effective Hamiltonian, which includes the $2^{\rm nd}$-order weak $\Delta b = 2$ transition, is diagonalized in the mass eigenbasis, obtained by solving

$$H|B_{\rm H,L}\rangle = E_{\rm H,L}|B_{\rm H.L}\rangle \tag{20}$$

where the subscript H and L denote the "heavy" and "light" eigenstates and the effective Hamiltonian can be written as

$$\begin{aligned} H &= \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix} \\ &= \left(M - \frac{i}{2}\Gamma\right) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & 0 \end{pmatrix} \end{aligned} \tag{21}$$

where in the last line CPT symmetry is used to write $M = M_{11} = M_{22}$ and $\Gamma = \Gamma_{11} = \Gamma_{22}$. The values of M and Γ are determined by the quark masses and the strong and electromagnetic interactions. The last term induces $\Delta b = 2$ transitions and is responsible for flavor oscillations. The dispersive (M_{12}) and absorptive (Γ_{12}) parts correspond to virtual and real intermediate states, respectively. The time evolution of a state that is a pure B^0 at $t = 0$ is given by

$$\begin{aligned} |B^0(t)\rangle &= e^{-(\Gamma/2 - iM)t} \left\{ \cos\frac{\Delta M t}{2} \cosh\frac{\Delta\Gamma t}{4} - i \sin\frac{\Delta M t}{2} \sinh\frac{\Delta\Gamma t}{4} \right\} |B^0\rangle \\ &+ \tfrac{q}{p}\, e^{-(\Gamma/2 - iM)t} \left\{ i \sin\frac{\Delta M t}{2} \cosh\frac{\Delta\Gamma t}{4} - \cos\frac{\Delta M t}{2} \sinh\frac{\Delta\Gamma t}{4} \right\} |\overline{B}^0\rangle \end{aligned} \tag{22}$$

where (ignoring CP violation for the moment) $|p|^2 + |q|^2 = 1$, $M = \frac{1}{2}(M_{\rm H} + M_{\rm L})$, $\Gamma = \frac{1}{2}(\Gamma_{\rm H} + \Gamma_{\rm L})$, $\Delta m = \frac{1}{2}(M_{\rm H} - M_{\rm L})$, and $\Delta\Gamma = \frac{1}{2}(\Gamma_{\rm H} + \Gamma_{\rm L})$. In the B system we always have $\Delta\Gamma \ll \Gamma$, since the branching fraction to flavor-neutral intermediate states (with quark content $c\overline{c}d\overline{d}$ or $u\overline{u}d\overline{d}$) is $\mathcal{O}(1\%)$, and only these can contribute to $\Delta\Gamma$. The formula above simplifies in this approximation. No such constraint applies to $\Delta m/\Gamma$, since virtual intermediate states contribute. In fact, the large top quark mass breaks

the GIM mechanism that would otherwise cancel this FCNC and enhances Δm.

The dominant diagrams responsible for $B^0\overline{B}^0$ oscillations in the SM are W-t box diagrams. While the short-distance physics in these diagrams can be calculated perturbatively, there are also non-perturbative matrix elements that enter the width. The standard OPE expression for Δm is[16]

$$\Delta m_q = \frac{G_F^2}{6\pi^2}\eta_B(f_{B_q}^2 \hat{B}_{B_q})M_W^2 S_0(m_t^2/M_W^2)\,|V_{tq}|^2 \tag{23}$$

where η_B is a perturbative QCD correction, f_B is the B meson decay constant, $\hat{B}_B$ is the "bag factor", and q can be either d or s. The term S_0 is a known function of m_t^2/M_W^2 that has a value of ~ 2.5 for the measured masses. The uncertainty in $f_B^2\hat{B}_B$ produces a $\sim 30\%$ uncertainty in extracting $|V_{td}|$ or $|V_{ts}|$ by comparing Δm_q with measured values.

Many methods have been used to study $B^0\overline{B}^0$ oscillations. The most sensitive is the dilepton charge asymmetry[17] as a function of the B^0 decay time (or, in the case of the B factories, the decay time difference between the two B mesons). The current world average[1] for the oscillation frequency is $\Delta m_d = (0.489 \pm 0.008)\,\mathrm{ps}^{-1}$. Improvements in calculations of $f_B^2\hat{B}_B$ are needed in order to improve the impact this measurement has on constraining the unitarity triangle.

B_s^0 oscillations cannot be studied in $\Upsilon(4S)$ decays. The 0^{th}-order expectation for Δm_s is $\Delta m_s = |V_{ts}/V_{td}|^2\Delta m_d \sim 15\,\mathrm{ps}^{-1}$, but the numerical value is not very precise due to uncertainties in the CKM elements. The large value of Δm_s implies very rapid oscillations and makes the measurement of Δm_s challenging. Note that evidence exists for B_s^0 flavor change;[18] it is consistent with being maximal. However, at present there are only lower limits[1] on Δm_s from experiments at LEP and from CDF. Similar uncertainties in $f_B^2\hat{B}_B$ arise when comparing the theoretical and measured values of Δm_s as for Δm_d. However, the theoretical uncertainty on the ratio $\Delta m_d/\Delta m_s$ is smaller, as some of the uncertainties in the ratio $(f_{B_s}^2\hat{B}_{B_s})/(f_{B_d}^2\hat{B}_{B_d})$ cancel; this is an active area of theoretical investigation.[21] A measurement of Δm_s will provide significant information in constraining the unitarity triangle.

6. CP violation

The CP operation takes particle into anti-particle, admitting an arbitrary (and unobservable) phase change:

$$\mathrm{CP}|X\rangle = e^{+2\theta_{\mathrm{CP}}}|\overline{X}\rangle; \quad \mathrm{CP}|\overline{X}\rangle = e^{-2\theta_{\mathrm{CP}}}|X\rangle \tag{24}$$

CP violation was discovered[22] in K_L^0 decays to $\pi\pi$ final states in 1964. At the time there was no known mechanism that could accommodate this observation. Such a mechanism was introduced by Kobayashi and Maskawa[23] in 1973. They postulated the existence of a third generation of quarks, noting that the 3×3 unitary matrix describing the mixing of quark weak and mass eigenstates retains one phase that cannot be removed by rephasing the quark fields. The presence of this phase allows for CP to be violated in reactions involving two or more interfering amplitudes. The first particle of the third generation was discovered the very next year[24] and the first quark of the third generation[25] three years thereafter. The mechanism uncovered by Kobayashi and Maskawa in fact *requires* CP violation in the absence of special values of the non-trivial phase or other fine-tuning conditions, like quark mass degeneracies or the vanishing of a CKM angle.

CP violation has been observed more recently in the B system[26,27,28], and appears to conform to the expectations of the KM picture. It remains to be determined, however, if the KM mechanism is the sole source of the CP violation seen in the K and B systems.

An invariant measure of the size of the CP violation in the CKM matrix is given by the Jarlskog invariant[29]

$$J = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta \simeq A^2\lambda^6\eta \tag{25}$$

where c_{ij} and s_{ij} are shorthand for the cosine and sine of the angle θ_{ij}, and A, λ and η are the parameters of the Wolfenstein parameterization. The maximum value of J in any unitary 3×3 matrix is $(6\sqrt{3})^{-1} \sim 0.1$; the value in the CKM matrix is $\sim 4 \times 10^{-5}$, which underlies the statement that CP violation in the SM is small.

6.1. *CP violation in B decay*

The CP violation in the SM is the result of a phase, and is therefore only observable in processes involving interfering amplitudes. The mechanisms for generating this interference in B decays fall into three classes: CP violation in flavor mixing, CP violation in interfering decay amplitudes, and CP violation in the interference between flavor mixing and decay.

The CP violation first observed in the neutral K system arises in flavor mixing. The mass eigenstates in the K system are not quite eigenstates of CP, and can be described as having a small component with the opposite CP. This type of CP violation arises because of the interference between the on-shell and off-shell $\Delta s = 2$ transitions between K^0 and $\overline{K}^0$. In the B

system the smallness of $\Delta\Gamma/\Gamma$ (i.e. the on-shell transition width) suppresses this source of CP violation (in constrast to the neutral K system, where $\Delta\Gamma_K \simeq -2\,\Delta m_K$).

CP will be violated in $B^0\overline{B}^0$ mixing only to the extent that $|q/p| \neq 1$ (see eq. 22):

$$\left|\frac{q}{p}\right|^2 = \frac{\langle \overline{B}^0|H_{eff}|B^0\rangle}{\langle B^0|H_{eff}|\overline{B}^0\rangle} = \left|\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right| \neq 1 \tag{26}$$

A manifestation of this CP violation would be an asymmetry in the semileptonic decays,

$$A_{\rm CP} = \frac{\Gamma(\overline{B}^0 \to \ell^+\nu X) - \Gamma(B^0 \to \ell^-\overline{\nu} X)}{\Gamma(\overline{B}^0 \to \ell^+\nu X) + \Gamma(B^0 \to \ell^-\overline{\nu} X)} \tag{27}$$

which is proportional to $(1-|q/p|^4)/(1+|q/p|^4) = \Im(\Gamma_{12}/M_{12})$. In the SM this asymmetry is $\mathcal{O}(10^{-4})$ and cannot be calculated with precision due to large hadronic uncertainties in the determination of Γ_{12}.

Interference between competing decay amplitudes can also render CP violation observable in the SM. This is known as direct CP violation, and was first seen[30] as a non-zero value of ϵ'/ϵ in neutral K decays. In direct CP violation $|\overline{A}_{\overline{f}}/A_f| \neq 1$, implying the decay rate for $\overline{B} \to \overline{f}$ is not the same as for $B \to f$. All unstable particles (not just neutral mesons) admit this type of CP violation in principle. The decay rate asymmetry is

$$A_{\rm CP} = \frac{\Gamma(B \to f) - \Gamma(\overline{B} \to \overline{f})}{\Gamma(B \to f) + \Gamma(\overline{B} \to \overline{f})} \propto 2|A_1||A_2|\sin\delta\sin\phi \tag{28}$$

where A_1 and A_2 are competing decay amplitudes that lead to the same final state f, δ is the difference between the strong interaction phases of the two amplitudes, and ϕ is the difference between the weak phases (which include the CKM contribution). Once again, precise predictions of this type of CP violation are not available due to hadronic uncertainties in calculating the strong phase difference δ. It remains of interest, however, to search for decay modes where the weak phase difference $\phi \simeq 0$, in which case any $A_{\rm CP}$ which deviates significantly from zero is a sign of new physics.

The CP violation that arises due to the interference between $B^0\overline{B}^0$ mixing and decay is different, in that theoretical predictions nearly free from strong interaction uncertainties are possible. The interference arises between the decay $B^0 \to f_{\rm CP}$ and the sequence $B^0 \to \overline{B}^0 \to f_{\rm CP}$. Clearly only those final states that are eigenstates of CP can contribute. This type of CP violation can be present even when $|q/p| = 1$ and $\overline{A}_{\overline{f}_{\rm CP}}/A_{f_{\rm CP}} = 1$.

The quantity

$$\lambda_{f_{\rm CP}} = \eta_{f_{\rm CP}} \frac{q}{p} \frac{\overline{A}_{\overline{f}_{\rm CP}}}{A_{f_{\rm CP}}} \tag{29}$$

where $\eta_{f_{\rm CP}} = \pm 1$ is the CP eigenvalue, is independent of phase conventions and contains the information on CP violation. Since the interference is mediated by mixing, the CP asymmetry has a characteristic time dependence:

$$\begin{aligned} A_{\rm CP}(t) &= \frac{\Gamma(\overline{B}^0(t) \to f_{\rm CP}) - \Gamma(B^0(t) \to f_{\rm CP})}{\Gamma(\overline{B}^0(t) \to f_{\rm CP}) + \Gamma(B^0(t) \to f_{\rm CP})} \\ &= \frac{1 - |\lambda_{f_{\rm CP}}|^2}{1 + |\lambda_{f_{\rm CP}}|^2} \cos \Delta m_B t + \frac{2\Im \lambda_{f_{\rm CP}}}{1 + |\lambda_{f_{\rm CP}}|^2} \sin \Delta m_B t \end{aligned} \tag{30}$$

The coefficient of the $\cos \Delta m_B t$ term is a measure of direct CP violation (or CP violation in mixing, but this is always small in the B system). For decay modes dominated by a single decay amplitude it vanishes. The coefficient of the $\sin \Delta m_B t$ term is then a pure phase. This phase can be related to angles in the unitarity triangle with very little theoretical uncertainty. This is why the study of CP violation in neutral B decays has attracted so much attention. It's worth seeing how this comes about.

The ratio q/p is given by $\sqrt{M_{12}^*/M_{12}}$ (ignoring the small contribution of Γ_{12}), where $M_{12} \propto V_{tb}^* V_{td} e^{i(\pi - 2\theta_{\rm CP})}$, so $q/p = e^{2i(\theta_{\rm CP}+\theta_M)}$ is a pure phase. θ_M is the phase difference coming from CKM elements and $\theta_{\rm CP}$ is the arbitrary phase change in the CP transformation. The decay amplitudes are

$$A_{f_{\rm CP}} = \langle f_{\rm CP}|H|B^0\rangle = |A| e^{i(\delta + \phi_D)} \tag{31}$$

$$\overline{A}_{f_{\rm CP}} = \langle f_{\rm CP}|H|\overline{B}^0\rangle = \eta_{\rm CP} e^{-2i\theta_{\rm CP}} |A| e^{i(\delta - \phi_D)} \tag{32}$$

giving $\overline{A}_{f_{\rm CP}}/A_{f_{\rm CP}} = \eta_{\rm CP} e^{-2i(\theta_{\rm CP}+\theta_D)}$, where θ_D is the weak phase associated with the decay. Note that the dependence on the strong phase δ cancels in the ratio of decay amplitudes. Putting this together, $\lambda_{f_{\rm CP}} = \eta_{\rm CP} e^{2i(\theta_M - \theta_D)}$. This clean relation holds for B^0 decays to CP eigenstates that proceed via a single decay amplitude. Channels involving interfering decay amplitudes in general result in a mix of direct CP violation and CP violation in the interefence of mixing and decay, and in these cases the correspondence with unitarity triangle angles is not free from hadronic uncertainties.

6.2. *Measuring Unitarity Angles with CP Asymmetries*

6.2.1. $\sin 2\beta$

The "golden mode" for studying CP violation in B decays is $B^0 \to J/\psi K_S^0$. In this case the decay is dominated by the tree diagram with internal $W \to c\overline{s}$ emission, leading to $(\overline{b}d) \to \overline{c}W^+d \to (\overline{c}c)(\overline{s}d)$. This is not a flavor-neutral state at the quark level, but becomes so at the hadron level through K^0 mixing. For this decay one finds

$$\lambda_{f_{\rm CP}} = \eta_{\rm CP}\left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right)_{\rm B\ mixing}\left(\frac{V_{cs}^*V_{cb}}{V_{cs}V_{cb}^*}\right)_{\rm decay}\left(\frac{V_{cd}^*V_{cs}}{V_{cd}V_{cs}^*}\right)_{\rm K\ mixing} \quad (33)$$

$$\Im(\lambda_{f_{\rm CP}}) = \eta_{\rm CP}\sin 2\beta \quad (34)$$

with very little theoretical uncertainty. The CP eigenvalue is -1 (for $B^0 \to J/\psi K_L^0$ it is +1). This decay mode is also favorable experimentally. The product branching fraction $\mathcal{B}(B^0 \to J/\psi K_S^0)\mathcal{B}(J/\psi \to \ell^+\ell^-) \simeq 5 \times 10^{-5}$ is well within the reach of B factories, and the final state includes a lepton pair, enabling excellent background suppression.

The experimental determination of $\sin 2\beta$ involves three key elements:

- The reconstruction of the CP eigenstate, $J/\psi K_S^0$. This requires good momentum and energy resolution for charged particles and photons.
- The determination of the b quark flavor of the recoiling B meson at the time of its decay. This requires good particle identification in order to cleanly identify the charged kaons and leptons that are used to infer the b quark flavor.
- The determination of the difference between the decay times of the B decay to the CP eigenstate and the recoiling B. This requires an asymmetric collider to boost the pair of B mesons along the beam direction and excellent vertex resolution to extract the spatial distance between the decay points of the two B mesons.

The B factories have been optimized to make this measurement. Given a sample of B decays to CP eigenstates, a determination of the flavor of the recoiling B, and the time difference between the decays, one can form the CP asymmetry

$$\begin{aligned} A_{\rm CP}(\Delta t) &= \frac{N(B^0) - N(\overline{B}^0)}{N(B^0) + N(\overline{B}^0)} \\ &\simeq (1-2w)\sin 2\beta \sin \Delta m_B \Delta t \quad . \end{aligned} \quad (35)$$

The asymmetry is based on the flavor assignment (B^0 or $\overline{B}^0$) of the recoiling B, w is the probability of incorrectly determinating the flavor of the recoiling B, and Δt is the time difference between the two B decays. One must determine the dilution $(1-2w)$ in order to extract $\sin 2\beta$. Since the underlying asymmetry is an oscillatory function, its amplitude is also diminished by the imperfect resolution on the decay time difference. Both of these effects (flavor mis-tagging and Δt resolution) can be controlled by considering a related asymmetry formed using fully reconstructed B^0 decays to flavor eigenstates (like $B^0 \to D^{*-}\pi^+$) in place of the CP eigenstate sample:

$$\begin{aligned} A_{\text{mixing}}(\Delta t) &= \frac{N(\text{mixed}) - N(\text{unmixed})}{N(\text{mixed}) + N(\text{unmixed})} \\ &\simeq (1-2w)\, \cos \Delta m_B \Delta t \end{aligned} \tag{36}$$

The flavor eigenstate sample has much higher statistics than the CP eigenstate sample, ensuring precise determinations of w and the Δt resolution. In practice the dilution factor at the B factories is $(1-2w) \sim 0.25$.

The BaBar and Belle experiments first observed non-zero CP violation[26] in 2001 and have now measured $\sin 2\beta$ $(\sin \phi_1)$ with good precision.[27,28]

$$\sin 2\beta = 0.719 \pm 0.074 \pm 0.035 \quad \text{(Belle)} \tag{37}$$

$$\sin 2\beta = 0.741 \pm 0.067 \pm 0.034 \quad \text{(BaBar)} \tag{38}$$

The systematic errors are dominated by the statistics of the auxiliary samples used to evaluate them, and should continue to fall with increasing luminosity. Figure 2 shows the CP asymmetry observed in BaBar for both the $J/\psi K^0_S$ and $J/\psi K^0_L$ modes. The K^0_L mode is reconstructed by using the position of the K^0_L interaction in the calorimeter or muon system and the known B energy to estimate the K^0_L 4-vector and form the invariant mass of the $J/\psi K^0_L$ pair. While the background is higher than for the $J/\psi K^0_S$ sample, it's satisfying to see a CP asymmetry of the opposite sign!

The measurements of $\sin 2\beta$ now give precise constraints in the $\overline{\rho} - \overline{\eta}$ plane (see Figure 1).

6.2.2. $\sin 2\alpha$

There is substantial effort at present on determination of the angle α (ϕ_2). This angle is accessible in $b \to u$ transitions leading to $u\overline{u}d\overline{d}$ final states, e.g. $B^0 \to \pi^+\pi^-$. In contrast to the golden mode, however, there are both tree and penguin decay amplitudes that contribute appreciably to these

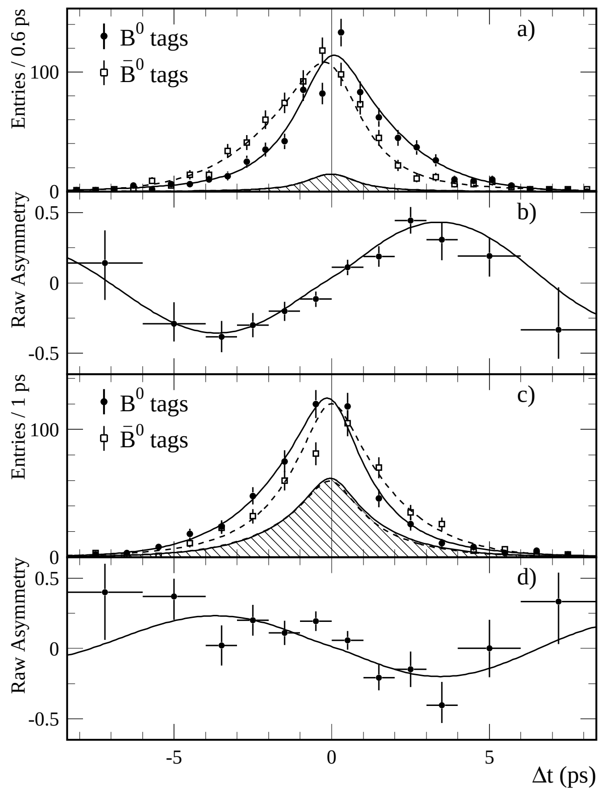

Figure 2. The distributions of Δt for events with recoiling B mesons tagged as B^0 and $\overline{B}^0$ (a) and the CP asymmetry (b) for $J/\psi K_S^0$ decays. The same distributions are shown in (c) and (d) for $J/\psi K_L^0$ decays. This figure is taken from Ref. [27].

decays, rendering the precise determination of α more difficult. The experimental situation is also less favorable for several reasons: the very small branching fractions[31,32] ($\mathcal{B}(B^0 \to \pi^+\pi^-) \simeq 5 \times 10^{-6}$); the higher backgrounds, primarily from the underlying $e^+e^- \to q\overline{q}$ continuum events; and the difficulty in distinguishing $B^0 \to \pi^+\pi^-$ decays from the more numerous $B^0 \to K^+\pi^-$ decays. The amplitudes of the underlying penguin and tree diagrams can be constrained[33] by measuring the branching fractions of the isospin-related channels $B^0 \to \pi^+\pi^-$, $B^0 \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$. This is, however, challenging; at present[35] only upper limits exist on $\mathcal{B}(B^0 \to \pi^0\pi^0)$. Other approaches[34] to limiting the uncertainty due to the penguin amplitudes have been proposed; this is an active area of investigation. In practice

one measures the coefficients $C_{\pi\pi}$ and $S_{\pi\pi}$ of the cosine and sine terms[c] in eq. 30. The coefficienct of the sine term would be $\sin 2\alpha$ in the absence of penguin contributions.

The current measurements of these coefficients in BaBar and Belle differ by about 2.5σ, and lead to rather different interpretations on the evidence for direct CP violation in $B^0 \to \pi^+\pi^-$ decays:[32,36]

$$S_{\pi\pi} = +0.02 \pm 0.34 \pm 0.05 \quad \text{(BaBar)} \tag{39}$$

$$S_{\pi\pi} = -1.23 \pm 0.41 \; {}^{+0.08}_{-0.07} \quad \text{(Belle)} \tag{40}$$

$$C_{\pi\pi} = -0.30 \pm 0.25 \pm 0.05 \quad \text{(BaBar)} \tag{41}$$

$$-A_{\pi\pi} = -0.77 \pm 0.27 \pm 0.08 \quad \text{(Belle)} \tag{42}$$

Note that the bound $S_{\pi\pi}^2 + C_{\pi\pi}^2$ must be satisfied by the true values of these coefficients. The Belle data are shown in Figure 3.

6.3. *CP Asymmetries in B_s^0 Decays*

The B_s^0 system can also be used to study CP violation. However, B_s^0 production is suppressed relative to B_d^0, implying smaller signals and higher backgrounds. The outlook for studying B_s^0 at threshold e^+e^- machines is not good. However, dedicated experiments at high energy hadron colliders are expected to contribute significantly in this area.

The presence of spectator s quark makes a different set of unitarity angles accessible in B_s^0 decays. The rapid oscillation term ($\Delta m_{B_s^0} \sim 30 \Delta m_{B_d^0}$) makes time resolved experiments difficult, but not impossible. The width difference between the B_s^0 mass eigenstates may be exploited as well.

7. Rare decays

Rare B decays offer a window on new physics. A particularly fruitful place to look for new physics is in FCNC decays, which are highly suppressed in the Standard Model. New physics can enhance these rates substantially; see Refs. [37] and [20] for overviews of this subject. The first $b \to s$ FCNC decays observed,[38] namely $b \to s\gamma$, still provide the best limit on the mass of a charged Higgs boson. The penguin decays $b \to s\gamma$, $b \to s\ell^+\ell^-$ ($\ell = e$ or μ) and $b \to s\nu\overline{\nu}$ are areas of active investigation at the B factories. The

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[c]This is another unfortunate case of notational differences. BaBar and Belle use different conventions (related to the inclusion or not of the CP eigenvalue in the definition of the coefficients) resulting in $C_{\pi\pi}^{\mathrm{BaBar}} = -A_{\pi\pi}^{\mathrm{Belle}}$.

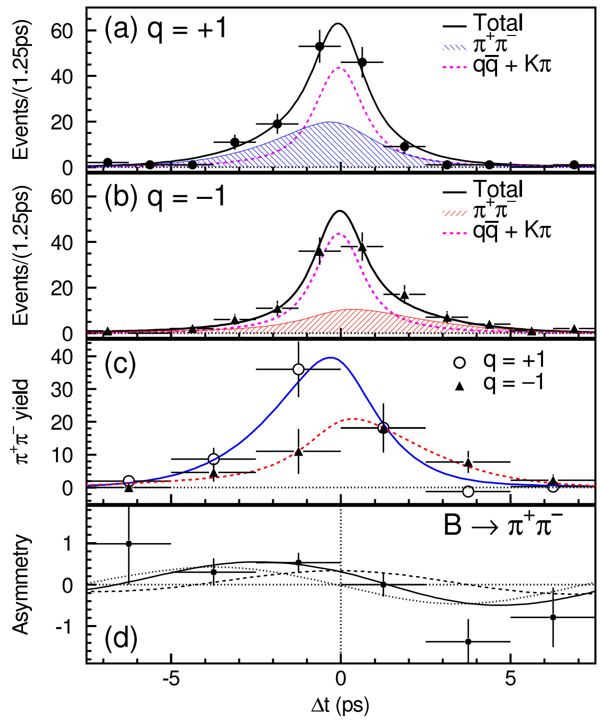

Figure 3. Distributions of $B^0 \to \pi^+\pi^-$ events versus Δt for B^0 tags (a) and $\overline{B}^0$ tags (b). The background-subtracted distributions and the asymmetry are shown in (c) and (d). This figure is taken from Ref. [36].

CKM-suppressed modes, with s replaced by d, have not yet been observed; their measurement will allow clean determinations of the ratio $|V_{td}|^2/|V_{ts}|^2$.

The branching fraction for $b \to s\gamma$ is now measured[38] and predicted[39] with good precision:

$$\mathcal{B}(b \to s\gamma) = (3.3 \pm 0.4) \times 10^{-4} \quad \text{experiment} \tag{43}$$

$$= (3.29 \pm 0.33) \times 10^{-4} \quad \text{theory} \tag{44}$$

The B factories have recently observed the first evidence for FCNC decays involving Z penguin and W box diagrams. Belle has measured[40] the inclusive decay $\mathcal{B}(b \to s\ell^+\ell^-) = (6.1 \pm 1.4\,^{+1.4}_{-1.1}) \times 10^{-4}$ and the combined BaBar[41] and Belle[42] measurements give $\mathcal{B}(B \to K\ell^+\ell^-) = (7.6 \pm 1.8) \times 10^{-7}$. These are compatible with SM expectations.[37] The sensitivity will improve with

increasing luminosity and the forward-backward asymmetry of the lepton pairs will be measured, providing stringent constraints on new physics.

The related decay $b \to s\nu\overline{\nu}$ is also of interest. It is very clean theoretically, and is sensitive to all three generations. The SM rate is also higher than for $b \to s\ell^+\ell^-$ due to the larger couplings of neutrinos to the Z. The experimental signature, which includes at least two missing particles, makes searches for these modes also sensitive to exotic, non-interacting particles (e.g. the lightest supersymmetric particle). The best existing limit is on the mode $B^+ \to K^+\nu\overline{\nu}$, where BaBar has a preliminary result[43] of $\mathcal{B}(B^+ \to K^+\nu\overline{\nu}) < 9.4 \times 10^{-5}$ at 90% c.l. The sensitivity of these searches is improving as data sets increase.

8. Summary

The B factory program has had a very fruitful beginning—and it is still the beginning. The next few years will bring significant advances in our knowledge of flavor physics.

CP asymmetries in B decays have been observed, are large and will be observed in many modes in the coming years. Precision studies of B decays and oscillations provide the most significant source of information on three of the four CKM parameters and are beginning to provide stringent tests of the Standard Model. The interplay of theory and experiment is vibrant and necessary in order to extract precision information on the matrix elements $|V_{ub}|$ and $|V_{cb}|$ and on the angles α and γ of the unitarity triangle. Rare B decays offer a good window on new physics due to the large top quark mass, and the sensitivity to these decays is improving rapidly. B hadrons are also a laboratory for studying QCD at large and small scales, and a theoretical framework has been developed to make precise predictions and suggest new measurements to test the soundness of the framework.

The constraints of time have dictated an abbreviated treatment of a number of the topics covered here and have precluded the inclusion of others. The space devoted to the wealth of B physics measurements made at non-B factory facilities has been minimal; I can only refer the interested reader to broader reviews of B physics, e.g. as given in Ref. [44].

The field of flavor physics is vibrant at the start of the 21$^{\text{st}}$ century. The B factories and neutrino experiments, in discovering CP violation in B decays and neutrino oscillations, have produced the most significant discoveries since the LEP/SLC program. These same two fields will probe deeper into the mysteries of flavor oscillations and CP violation throughout

this decade. Flavor physics is becoming precision physics!

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